NEW PRODUCT DECISIONS: INFORMATION DISCOUNTING AND PRODUCT SELECTION

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INTRODUCTION

The advantages of introducing new products are widely recognized by businessmen. Successful new products may generate substantial rewards for the innovating firm and insure a healthy growth rate for the firm. Although the advantages are great, the risks of new product introduction are high. The failure rate for new products has been estimated in various studies. The most conservative of these suggests that one-third of new products introduced are not financially successful.[1]

A New Product Planning System

Several model building approaches have been proposed to help reduce this high failure rate by increasing the quality of new product decisions. The modeling that has been done can be viewed within a total product planning system. This system has four steps: (1) search, (2) screening, (3) analysis, and (4) implementation. The process begins with a search effort designed to generate a large number of potential projects by exhausting the creative idea sources inside and outside the company. Pessemier has developed a stochastic model to aid in allocating resources in this search stage.[2] The ideas resulting from the search program are then screened to remove the unsuitable or unattractive proposals. O'Meara[3] proposed an early rating system approach that has been extended by Burton Dean[4] and others working in the area of R & D project selection.[5] The most recent extensions in the new product area have been made by Freimer and Simon.[6] The limited number of projects that pass through the screening stage

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are then subjected to detailed analysis. To assist in this analysis stage, two models, DEMON[7] and SPRINTER[8], have recently been designed. Both these models attempt to determine whether the project should be rejected (a NO decision), whether more study should be carried out (an ON decision), or whether the product should be accepted (a GO decision). If a NO decision is indicated, the project is rejected. If an ON decision is indicated, a study is carried out to gain additional information. A sequence of ON decisions represent a project's movement down an information network. After each study a new evaluation is carried out and a new GO, ON, or NO decision is made. If the project has been accepted (GO decision), implementation is begun. PERT planning models are useful at this stage. Implementation models designed to utilize early market data in predicting the ultimate success of a product have also been developed. Fourt and Woodlock have developed a penetration model[9] and recently Massy has designed a stochastic model called STEAM[10] to make these predictions. This paper is an attempt to extend the model building efforts at the analysis stage of the new product decision process.

<u>The New Product Decision at the Analysis Stage</u>

The input to the analysis stage is a set of new product proposals that have passed through the elimination grid of the screening process. At the analysis stage the best sub-set of proposed projects must be selected. A sub-set may have to be accepted since financial, managerial, or production constraints may exist in a given time period. The analysis of this selection problem and the development of a mathematical model to aid in solving this problem are the purposes of this paper.

Before a selection criterion can be applied to the set of proposed projects, each project should be brought to its most profitable posture. The best mix of marketing, production, and financial variables should be found.

The DEMON[11] and SPRINTER[12] models each attempt to describe the best marketing mix for a new product. In accomplishing this, SPRINTER for example, uses an estimate of the sales for the product over time -- the life cycle -- as its reference input. It then utilizes response functions that measure the proportionate changes in the reference sales level in each year or the result of changing a marketing parameter. When the response functions are multiplied times the life cycle values, the sales implications of various marketing programs are generated. These estimatates are reduced by a competitive term that reflects the firm's market share to estimate the new product sales. When this sales level is combined with the firm's cost structure, a profit equation is generated. This profit is then optimized, subject to the firm's production, financial, and managerial constraints, by an iterative search routine which specifies the best marketing mix for the new product in each year of the planning period. If the product displays demand or cost inter-dependence with existing products, the change in the total firm's profit or the "differential profit" is used as a criterion in the optimization. The best estimate of differential profit is combined with the uncertainty surrounding the product to describe the most desirable state for the product. The uncertainty estimate is obtained by aggregating the confidence estimates about the reference sales estimate, response functions, competitive effects, and costs to determine the variance of the new product profit distribution. If the new product is inter-related to the old products, the "differential uncertainty" or the variance of the differential profit distribution is utilized.

The estimates of risk and return generated by analysis models can be used to specify the GO, ON, or NO decision for the product. A criterion such as maximizing profit subject to a constraint on the probability of achieving a minimum rate of return on investment may be used.[13]

If each of a set of proposed projects is considered sequentially and a GO, ON, or NO decision is made for each, there is no selection problem. The problem has been avoided by making an independent decision for each project. This sequential procedure has been adopted in existing analysis modeling, but it may not be satisfactory since the final set of accepted products will be sensitive to the order in which the projects were proposed.

It is desirable to consider all the projects as a group since it may not be feasible to adopt all the projects that are found acceptable. The risk-return profiles of each product should be compared after each project has been placed in its most profitable position. But even this procedure could be biased since the selection would favor those products that have received the most study and have proceeded the farthest down the information network. These projects will have little uncertainty associated with them, while projects that have not progressed as far down the information network will be subject to greater uncertainties of estimation and therefore appear to be in a less attractive risk-return position.

For example, if the firm were to choose between a new videotape recorder that had been test marketed and a three dimensional home movie projector that had not been consumer tested, it might appear that the risk-return position of the tape recorder is preferable to that of the movie projector. A market test of the projector, however, could revise its risk-return position so that it is preferred to the tape recorder. The selection should be made on the basis of the best information posture of the product. What is needed is a model to consider both the profit and potential information characteristics of the new product.

A two stage modeling approach is appropriate. In the first stage the profit attributable to the product is maximized by the specification of the

optimum values for the controllable variables of the product. The second stage of the analysis is to consider the effects of future studies on the product. The value of information gained from future studies should be attributed to the product. The maximum value of information would be determined by finding the optimum number, size, and sequence of studies to be carried out concerning the new project. This maximum value of information could then be combined with the current risk-return posture of the proposal to obtain its best overall standing. These revised evaluations would supply a reasonable basis for selection.

The first stage analysis has been modeled by DEMON and SPRINTER. The purpose of this paper is to develop a model for the second stage of the analysis and to discuss methods of selecting the best set of new products at the analysis stage of the new product decision.

INFORMATION DISCOUNTING AND THE INFORMATION NETWORK

The basic problem in the second stage of the new product analysis is to find the best sequence of information gathering studies for the proposed project. The determination of this optimal transversal of the information network and imputation of the gains from this study sequence will be called "information discounting." Information discounting is used as a descriptor because the process is one of determining the present value of future information studies. This is somewhat analogous to financial discounting of cash flows, so information discounting is utilized as the name of this process.

Bayesian Methods of Information Discounting

One approach to information discounting is by the use of Bayesian pre-posterior analysis. Pre-posterior analysis is designed to yield the expected value of the information generated by a study. In examining an information network, two methods of Bayesian analysis may be used: fixed mode or sequential mode. The fixed mode of analysis is based on considering a sequence of studies as one aggregate study while the sequential mode is concerned with examining the path to be taken at each study branch.

For example, a sequence of three information studies would produce a network with three nodes. At each node three study possibilities exist and after each study there is also the option to terminate the network. In the fixed mode of analysis, the expected value of information would have to be calculated for each possible path through the network. Assuming: (1) S_i states of nature and $P(S_i)$ prior probabilities for each stage, (2) Z_j possible results from the sequence of tests with associated

reliabilities $P(Z_j|S_i)$ for the sequence, (3) a payoff matrix R_{ki} associated with the final decisions GO (k=1) and NO (k=2) and states of nature S_i, the expected value could be calculated for each possible sequence.

The expected value of the sequence (EV_{SQ}) is the expected value of the returns of the decisions that result from each Z_j less the costs of carrying out the sequence. The costs are the out of pocket costs of the study sequence and the opportunity costs that may be occasioned by competitive losses due to delay in introducing the product.

The value of the sequence:

$$EV_{SQ} = \sum_j [P(Z_j)\cdot EV_j] \quad - \quad \text{COSTS}$$

$$P(Z_j) = \sum_i P(Z_j|S_i)\cdot P(S_i)$$

where EV_j is the expected reward if Z_j occurs and the best decision is made. It is the greatest of

$$\sum_i P(S_i|Z_j)\cdot R_{1i} \text{ and } \sum_i P(S_i|Z_j)\cdot R_{2i}$$

where

$$P(S_i|Z_j) = \frac{P(Z_j|S_i)\cdot P(S_i)}{\sum_i P(Z_j|S_i)\cdot P(S_i)}$$

The sequence to be selected would be the one which has the highest expected value of utility.

This Bayesian approach is highly attractive on a theoretical basis, but there are a number of problems that preclude its use in practical problems. The first problem concerns the inputs necessary to analyze a sequence. In the new product network, each study may be concerned with a different aspect of the product, such as cost, demand, or competition. To use the

fixed mode these heterogeneous experiments must be aggregated into one equivalent study. This aggregation may present problems. For example, the aggregate test results (Z_j) may be difficult to specify since each sub-experiment produces completely different magnitudes and units of results. Similar problems are confronted when conditional probabilities $P(Z_j|S_i)$ are to be estimated, since this value must represent one over-all conditional probability of observing Z_j for the aggregated sequence when each study is directed at a different new product characteristic.

An even more basic difficulty is in defining the units of the states of nature (S_i) for the aggregated sequence when each study is measuring a different characteristic.

The second difficulty with the fixed mode analysis is a computational one. Each sequence evaluation may demand a large amount of calculation since the number of S_i and especially Z_j will probably be large. Continuous distributions could be used to reduce the calculation burden if standard distributions were appropriate,[14] but a substantial amount of computation would still be required since the number of sequences to be considered could be large. Even with only four study possibilities, 64 sequences would have to be evaluated. The number of studies is

$$\sum_{i=1}^{n} {}_nP_i$$

where ${}_nP_i$ = the permutation of "n" things taken "i" at a time. This expands rapidly as the number and kinds of studies are increased and various sizes for each study are allowed. The last problem with the Bayesian approach is that the reward values (R_{ki}) may reflect a non-linear utility function. In this case the expected utility calculation would be more difficult and further burden the computational aspects of the problem.

The sequential mode of Bayesian pre-posterior analysis is not as limited by the problems of defining the terms of the analysis, but the computational problems are much greater than in the fixed mode. This problem arises because after each test result (Z_j) three decision alternatives are present: GO, ON, and NO. The ON rewards are not known until the final study has been carried out, so each Z_j branches to a new test which produces new test results, each of which branches to another test. In fact, the number of decision alternatives that must be evaluated is:

$$\sum_{i=1}^{n} T^i \; {}_nP_i$$

where

n = the number of tests

T = number of possible test results to be considered

This is a rapidly expanding number and imposes great computation and input generation demands upon the analysis. These considerations preclude the use of sequential mode of Bayesian analysis for most realistic new product networks unless heuristic procedures can be developed to reduce the computational burden. Although pruning procedures to reduce the size of the decision tree may be possible in some problems, it is difficult in an a priori basis to eliminate sequences in the new product networking problem.

The Bayesian value of information procedure is theoretically the best method of information discounting in a new product information network, but the problem of aggregating the studies into one study for fixed mode analysis and the computation problems produced by the combinatorial nature of the network in sequential analysis discount its use as a practical method of information discounting.

Other Approaches to Information Discounting

Dean and Hauser have developed a procedure for estimating the value of information for a special class of one stage network problems.[15] They have formulated a model which allocates a fixed research budget among "n" different technical approaches to achieving a project's objective. The criterion for allocation is the maximization of the probability of the project succeeding,assuming only one of the approaches must succeed in order to establish the project goals. This approach implicitly uses the probability of success as a measure of the value of information,and the allocation of the budget among the alternatives represents an information discounting method for the one stage information network. The question of sequencing does not arise since it is assumed that all the selected approaches will be carried out simultaneously and therefore there is only one final evaluation node.

Two other methods have been proposed by Charnes, Cooper, DeVoe and Learner to analyze information network effects. The first is based on the use of matrix algebra to represent the network.[16] This network representation is reasonable, but it is unable to encompass the complex non-linear information value gains that may occur in an actual new product network. These networking concepts were utilized by Charnes, et. al. in a revised formulation of the DEMON networking model.[17] This revision utilized chance constrained programming and although theoretically attractive, it is not feasible for networks with more than two nodes.[18]

A Heuristic Approach to Information Discounting of a New Product Proposal

There is a need for a practical and theoretically acceptable method of information discounting. This method must find the optimum path through

a multi-stage information network. This path will be specified by the number, size, and sequence of studies that place the new product proposal in its highest utility position.

Information studies may be viewed as serving two basic functions. The first is to reduce the uncertainty associated with the new product, and the second is to prevent the mistake of introducing an unsatisfactory product. These two functions can be reflected in the utility of a new product proposal. For the moment, let us assume the utility level at a point n in an information network is represented by a certainty equivalent of the form.

$$(1) \qquad U_n = \mu_n - a\sigma_n^2$$

where μ_n = the expected value of profits at point n, a = constant, and σ_n = standard deviation of profits at point n. This type of certainty equivalent has been used successfully by Freund[19] and Farrar[20] in the emperical analysis of portfolio selection problems.

The two functions of the information network can be reflected in this utility surrogate. The first function is to reduce the uncertainty associated with the project. This reduction could be measured by the change in variance of the profit distribution. This is reflected in a narrowing of the confidence intervals about the estimates of profit. If the expected value remains the same, the study has no costs, and standard deviation is reduced, the utility level is increased. If there is a cost associated with the study, this cost could be deducted from the expected value, and the new utility of profits could be determined by equation (1). In this way, increases in desirability of a project may be affected by a study that reduces the estimation uncertainty of the proposal.

The second function of a study is to prevent the mistake of introducing an unprofitable product. This can be measured in a particular study by establishing a project rejection level for values of the test outcome.[21] For example, an unsatisfactory test market results may be a sales level below some specified amount. This level implicitly includes the possibility of measurement error and the manager's ability to tolerate it in a rejection decision. In this way, a test can reject a proposal before full introduction and investment expenditures have begun. The savings that may result from a future study that prevents this mistake can be estimated by the probability of a value below the rejection level times the savings produced by not marketing an unsatisfactory product. This savings can be considered as increasing the expected value of the profit generated by the product. If the expected value increases more than the study costs, the utility level will increase when the standard deviation is constant.

The combination of a study that reduces uncertainty and prevents introducing a poor product can be visualized by a decrease in the variance of the distribution and a change in the expected returns. These mechanisms allow the assessment of the results of information studies on a new product proposal. This assessment supplies a theoretically acceptable approach to evaluating the results of an information network. The changes in the variance and expected value could be considered in other objective functions than the certainty equivalent. For example, the probability of achieving or exceeding the firm's target rate of return on investment could be used. This implies a "satisficing" behavior for the decision maker. If the new product cash flow is discounted at the firm's target rate of return,

the probability of achieving the target rate of return can be found by examining the distribution of the discounted cash flow profits generated by the product. The probability is measured by the area under the distribution curve and to the right of the investment required by the project. For the purposes of this model exposition, the certainty equivalent measure will be used, but this is not a necessary criterion for the model.

An example of a new product test study will clarify the proposed information value assessment procedure. If the firm could carry out a test market, it would be able to improve its estimation of the product's sales level. The test might also show very low sales levels for the product and cause a NO decision to be made. For example, suppose a five city test market would cost $100,000 and managers estimate on the basis of similar test markets that it could reduce the standard deviation of the sales estimate by twenty-five percent. Further let us suppose that management establishes a rejection level for the product where the sales are less than twenty-five percent of best sales estimate. These two sales effects must be imputed to the utility function. If the sales level were the only uncertain variable in the problem, the profit variance would be proportional to it, but if it were not, the change in the sales variance would have to be aggregated with other variances by analytical or Monte Carlo methods.[22] The rejection sales level can be translated into a profit by multiplying by the price and subtracting the relevant production costs. The new and old profit distributions are shown in Figure 1 assuming normal distributions. The savings the test may produce by early rejection is the probability of profit being less than the rejection level (2 million dollars) times the savings of early rejection which we may suppose is 4 million dollars

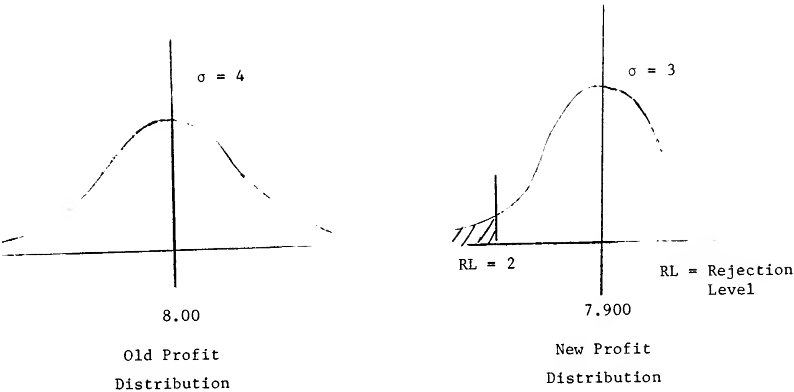


FIGURE ONE

PROFIT DISTRIBUTIONS

(see shaded area of Figure 1.) The increase in expected value for the test is \$91,000. The new utility for the project can be obtained by equation (1) and is 7.1 assuming $a = 1/10$, $[(8.09-.1) - .1(3)^2]$. In this case the increase in expected value of profits was not greater than the cost of the study. Even though the change in expected value was less than the cost of the study, the utility increased by about ten percent because of the decrease in the variance.

This information discounting method can be applied to the combinatorial problems of network analysis. The basic procedure requires only two inputs - the reduction in variance and rejection level for the test. The reduction in the variation could be subjectively estimated from experience with similar experimental designs. The rejection level could likewise be specified by managers based on subjective estimates of unsatisfactory performance. If these inputs were generated for each test in the sequence, the utility changes could be calculated. The inputs for each study in the sequence are the reduction in variance expected from this study conditional upon the other studies which have already been conducted in the sequence and the product rejection level of the new test. For example, if another test alternative was a pricing study, its effect on the proposal would be different if it were conducted after rather than before the test market.

The pricing study could be a simulated shopping trip[23] that tests alternated sales-price responses. Let us suppose for this example that if the pricing study were to be carried out first, management estimates that this type of test could reduce the standard deviation of the price response estimate by twenty-five percent and that the standard deviation of the price response accounts

for twenty percent of the sales deviations. The total sales deviation would then be reduced by five percent. In this study there probably would be a low rejection level since the study is directed at measuring response rather than overall product success. Let us assume the rejection sales level for this experiment is ten percent of the expected sales level and that this test costs $10,000. This experimental condition increases the value of the uncertainty equivalent by .26 units. If this test were to be carried out after the test market cited earlier,and assuming the percentage reduction in standard deviations produced by the pricing study increased by 20 percent as the result of the experience gained in the pricing study, the utility would increase only by .10 units. This reduction in the utility gain is due to the fact that the two studies overlap. The test market will already have reviewed projects that would have fallen below the rejection level of the pricing test and also the test market had already reduced some of the sales variance. This assumes that the test market is perfect in identifying values below the rejection level. If this is not true because of sampling effects in the first test, the second test might identify a project below the rejection level. This could be approximated by multiplying the probability of a type two error in the first test times the probability of rejection in the second test. For the purposes of this example, it is assumed that the first test has no type two error. If the test market would be carried out after the pricing experiment, and assuming the test market would now reduce the standard deviation by thirty percent, the test market study would increase the probability by .65 units. This is less than the .7 unit increase generated if it had been conducted first.

The order in which studies are conducted will affect the total value of a sequence. In the example above, if the price study were conducted before the market test, the total utility to both studies would be 7.31, while if the reverse order were used, only 7.20 would be recorded. The best sequence depends upon the interdependencies between the tests. Some studies may completely overlap others, while some studies may improve the results of future studies.

If the conditional inputs could be estimated for each study, the rewards for each study sequence could be specified. If the procedure used in the last paragraph is employed to generate conditional values, all sequences will be evaluated. This method of input generation may be very tedious, but economies of scale may be possible. For example, the marginal effects of a study may be obvious because of a complete or a constant proportional overlap between studies. If a number of degrees of a particular study are contemplated, the input may be generated for each degree by a transformation of one basic value. Finally the marginal effects of some studies will be independent of others. These considerations indicate marginal values for each study sequence may be feasibly generated.

The problem is now to find the best sequence and number of studies to include in the information network. This is a problem in combinatorial analysis. The permutations and combinations of the n study possibilities must be considered. The best approach to analyzing this network is to use a dynamic programming technique similar to that proposed by Held and Karp.[24] Branch and bound techniques[25] might be used, but they are not attractive here since the number of study combinations (the state space) increases as

the analysis proceeds down the network. Dynamic programming can encompass this difficulty.

Each study may be represented by one value. In this development it is the increase in utility when the study is used in a particular position in the information network sequence. The objective of the search of sequences is to find the one with the greatest increase in utility. The dynamic programming recursion relationship is

$$(2) \qquad T\Delta U_n(S_n) = \underset{d_n \varepsilon D_n}{\mathrm{MAX}} \left[f(d_n, S_n) + T\Delta U_{n+1}(G(d_n, S_n))\right]$$

where $S_{..} = G(d_{n-1}, S_{n-1})$

and $T\Delta U_N(S_N) = \underset{d_N \varepsilon D_N}{\mathrm{MAX}} f(d_N, S_N)$

$T\Delta U_N(S_N)$ = change in utility produced by final test in sequence

N = final test opportunity

$T\Delta U_n(S_n)$ = total change in utility produced by carrying out tests n to N given state S_n

S_n = permutation of studies completed before test n

d_n = test selected for use as the "nth" link in the information network

D_n = total set of test possibilities available for use at n

$f(d_n, S_n)$ = incremental utility gain generated by decision d_n and state S_n (see equation 1)

The dynamic programming policy based on this recursion would be the optimal sequence of studies for the proposed set of studies. Re-running the recursion for various N would access the effects of shortening or lengthening the information network. If a relevant set of kinds and sizes of studies

were included in the analysis, the optimal policy would prescribe the number of tests, size, and sequence of studies and impute the information gains of the network to the project. This is the desired information discounting model.

The model as formulated in equation (2) is based on the use of a certainty equivalent as a surrogate for utility. This can be adapted to reflect other goal structures. The decision rule of maximizing the expected value of profits subject to a constraint on probability could be used. The expected value of the sequence could be optimized in the dynamic programming recursion and then the value could be tested to see if it satisfies the probability constraint. If the highest expected value does not satisfy the constraint, other values could be tested in decreasing order of expected value. The first to satisfy the constraint would be optimal.

In this section, an information discounting model has been developed. It was based on the specification of two functions of new product studies: reducing uncertainty and preventing mistakes. These two functions were imputed to the utility associated with the project through the effects of the changes in the variance of the profits and increases in expected value. When these two inputs have been supplied for each study in each position in the sequence, a utility value can be calculated. The utilities are the inputs to a dynamic programming recursion which specifies the best sequence of studies and places the project in its optimal information position.

NEW PRODUCT SELECTION

When the information discounting model described in the previous section is applied to a set of new product proposals, the differences in the information state of each project are explicitly considered. If this discounting is done after the profit as a function of the firm's controllable variables has been maximized, each project is at its best profit and information position. The selection of the best sub-set of the total possible set of projects can be undertaken after these two stages of analysis.

Simple Selection Rules

The simplest solution to the selection problem would be to select the best product first and continue selecting in order of decreasing desirability until some constraint on the number of projects is met. The measure of merit utilized in this procedure would be important. If the criterion were to maximize the probability of the minimum rate of return, the projects would be selected in order of the probability. Projects would be accepted until the new product investment budget for the time period was expended or until the probability became unacceptable. If the goal is profit maximization with a constraint on the probability, projects would be selected in order of expected value until the budget was expended or until the probability constraint was violated. The most general criterion for the selection problem would be utility. If all the projects were plotted on a risk-return graph, utility levels might be used to rank projects. A certainty equivalent could be useful in ascertaining relative utility positions for each product.

This simple selection procedure ignores a number of the critical considerations that are present in the new product selection problem. First, the simple approach does not, in general, produce the best solution. This is because of its considerations of the constraints. It is possible that the sub-set consisting of the

individually most desirable projects may not most effectively use the firm's resources. Secondly, the simple procedure does not encompass investment and risk interdependencies that may exist within the set of new proposals. If differential profit and differential uncertainty measures were used in each proposal's evaluation, the old-new product interaction would be considered, but the new-new product interaction would not be encompassed. A number of operations research techniques can be of use in the analysis of these factors.

Selection of Independent Projects Under Certainty

First, examine the selection of projects subject to constraints, assuming each is independent and certain. Given a criterion such as profit, this becomes a simple mathematical programming problem. It is an integer problem if each project is to be considered an indivisible package.[26] This is not the case in the new product problem. The project can be varied in size. Advertising and pricing changes can change the profitability and investment commitment for the project. If the return to fractions of the project were linear, the selection would become a linear programming allocation problem. Usually the returns would be non-linear, so price-wise linear or other non-linear programming techniques would become relevant.

Selection of Independent Projects Under Risk

The certainty assumption can be modified so that risk considerations can be encompassed to some degree if expected values are used in the objective function. Dean, Chidambaram, and Palaski have developed a model of this type for R & D project selection.[27] In their model the expected value of reward is the profit generated by the project, given it is successfully developed, times the probability of successful development. Their selection criterion is based on maximizing the expected rewards subject to a constraint on research funds. The probability of success of each project is some

function of the research expenditure on the project, so the problem is usually non-linear and can be solved by some non-linear programming technique such as dynamic programming.

Another approach to the selection problem under risk is to use a certainty equivalent which imputes a utility value for the risk-return position of the product. Then the mathematical procedures discussed for the certainty case could be used.

<u>Selection of Independent Projects Under Risk with Covariance</u>

If projects display covariance but are independent investment projects, the techniques developed for portfolio selection are relevent. These techniques allow projects to be risk interdependent by the use of covariances,but do not comprehend the basic interdependencies generated by demand complementarity or substitutability that are reflected in the projects' expected value. Markowitz formulated the basic selection problem under these conditions as a two step procedure.[28] First, the total variation of the portfolio is to be minimized subject to a minimum required expected value of profit. This is a quadratic programming problem. The second step in the analysis is to repeat the quadratic program for various levels of minimum expected return. The output of these two steps is a line of "efficient" portfolios. The investor is then to designate the point on this line that offers him the greatest utility. Markowitz's approach has been modified by Farrar to be a direct maximization of utility by the use of a certainty equivalent.[29] Both of these approaches assume that projects are divisible and that constant returns are present. This assumption, however, has been relaxed by Weingartner[30] in an integer formulation of the selection problem based on earlier work by Cord[31].

One significant disadvantage of the portfolio models in the context of new product selection still exists. This is the inability to consider product

interdependencies produced by demand or cost complementarities or substitutability.

Selection of Interdependent Projects Under Certainty

Product interdependencies have been included in capital budgeting models when each project's returns are considered certain. Weingartner has developed in integer programming model which allows mutually exclusive and contingent investments to be considered.[32] Reiter has presented a generalized form of interdependency that allows partial degrees of interaction between pairs of projects.[33] The chief drawback of this model is that it assumes certainty.

Selection of Interdependent Projects Under Risk with Covariance

To simultaneously encompass the critical new product selection characteristics of non-linear returns to fractional projects, product interdependency, and covariance, direct search techniques must be utilized. These direct search techniques are based on heuristic rules that control the movement of the search and attempt to generate good and sometimes optimal techniques. Wilde and Beightler present a very fine discussion of the large number of search techniques in their book, Foundations of Optimization.[34]

A reasonable objective function for the generalized search procedure would be to maximize

$$(3) \qquad U = \sum_i X_i \mu_i + \sum_{\substack{i,j \\ i<j}} X_{i,j}\, \mu_{ij} + \sum_{\substack{i,j,k \\ i<j<k}} X_{i,j,k}\, \mu_{ijk} - a \sum_{i,j} X_i\, \sigma_{ij}\, X_j$$

Subject to:

$$\sum_i^n X_i M_i \leq M_T \qquad\qquad \sum_i^n X_i I_i \leq I_T$$

$$X_{i,j} = X_i X_j \qquad\qquad X_{i,j,k} = X_i X_j X_k$$

$$X_i = 0,\ 1$$

U = total utility of selected products as measured by uncertainty equivalent

μ_i = expected profit return for selecting project i

μ_{ij} = change in expected value of profit when projects i and j are selected

μ_{ijk} = incremental change in expected value of profit when projects i, j, and k are selected

σ_{ij} = covariance between project i and j

a = constant reflecting risk aversion

M_i = total commitment of trained managers necessary to fully administer product i

n = number of projects in total set of proposals

M_T = total number of trained managers available

I_i = investment required for full implementation of product i

I_T = total investment available for new products

This is a certainty equivalent approach that encompasses product interdependency and uncertainty in the framework of a constrained maximization. The first term of equation (3) is the total expected value generated by selecting a sub-set of projects assuming there are no more than ternary product independencies. The second term is the total variance of the profits multiplied times a risk aversion factor. If fractional project commitments were to be considered: (1) X_i would be non-integer, (2) μ_i, μ_{ij}, and μ_{ijk} would be non-linear functions of X_i, $X_{i,j}$, and $X_{i,j,k}$ respectively, (3) σ_{ij} would be a function of μ_i, μ_{ij}, and μ_{ijk}, and (4) the number of managers and investment might not be linearily related to X_i.

The maximization posed in equation (3) could be attached by a number of heuristic search techniques. One example of a search procedure in a related selection problem has been presented by Weingartner when he allowed uncertainty and simple interdependency to be considered in a capital budgeting problem.[35] This was based on Reiter's discrete optimizing procedure.[36] The best search technique to use for the optimization will be a function of the particular

response characteristics and the computational efficiency of various search techniques when applied to the particular selection problem.

The method to be used in deriving solutions to the new product selection problem is dependent upon the factors to be considered in the problem. First, the problem should be defined and then the techniques most appropriate to the problem should be chosen. If the products' expected values are independent a portfolio selection model is appropriate. If the projects show product interdependencies, but certainty may be assumed, integer capital budgeting models are applicable. If the products are interdependent and covariance is important, heuristic generalized search procedures are appropriate.

The application of a selection technique to the set of possible products will produce the best sub-set of new products. This sub-set of products may include some products that currently satisfy the firm's criteria for a GO decision and some that have satisfied these criteria by gains due to information discounting. Those that currently are in the GO condition may move on to implementation, although additional tests may still be carried out on the product during the pre-implementation period. The projects in the selected sub-set that fell in the ON condition before the information discounting would begin movement down the optimal information network path specified by the discounting model. As each study is completed the results would be re-evaluated and an updated GO, ON, or NO decision would be reached. The test results might indicate new relationships that would reflect the optimum mix of marketing, production, and financial variables. If this were true, the first stage analysis model would be rerun. The second stage information discounting model would be rerun if the selection decision were to be reviewed or if there was reason to believe the study sequence should be altered. This is an adaptive procedure. Selection decisions are made,

studies are carried out, and projects are reviewed. The marketing mix or information network path may be adjusted after each study as a result of the new information. The selection decision itself may be periodically reviewed as projects proceed down this network and new information is received.

NEW PRODUCT SELECTION: AN EXAMPLE

To clarify the information discounting model and selection procedure formulated in this paper, a simple example will be developed. For the purposes of this example, five proposals will be considered in the set of potential projects: (1) three dimensional home projector: (2) videotape recorder; (3) wall mounted thin TV; (4) TV telephone; and (5) programmable desk calculators. The projects will be assumed to have passed through a first stage analysis model such as SPRINTER which has maximized their profits at the given information state. The three dimensional home projector project will be processed through the information discounting model and then a selection procedure will be demonstrated assuming each product has attributed the present value of future information.

Information Discounting of a New Product Proposal

Suppose that five study alternatives are to be considered for the three dimensional home projector. For the purposes of this example, the criterion for discounting and selection will be maximizing the value of the certainty equivalent:

$U = \mu + (.1)\ \sigma^2$, where

U = utility

μ = mean profit level

σ = standard deviation of profits

The current state of the product will be defined by the following first stage analysis results:

investment = \$5 million

expected profit = \$8 million

standard deviation of profit distribution = \$4 million

utility $= 8 - .1\ (4)^2 = 6.40$

The five studies to be considered are:

(1) a five city market test

(2) a laboratory pricing experiment

(3) a one city market test

(4) an advertising response test

(5) a production cost study

The first and second studies were discussed in the example in the information discounting section of this paper. The five city test market improved the utility by .71 units, and the pricing experiment increased the utility .26 units.

The third alternative is a one city test market and it can be considered as measuring the same variables as the larger test market but with less accuracy. It is estimated to cost $25,000, and would reduce the standard deviation of the sales estimate by ten percent. A rejection level of ten percent of sales is set for this one city test market. This would increase the expected value of utility by .38 units. The advertising study is considered to reduce the standard deviation of the advertising response by twenty-five percent and a rejection level of ten percent of estimated sales is established. The advertising response contributes 20 percent of the standard deviation of sales. This study would increase the utility by .26 units. The cost study is considered to be a production test of the variable unit costs of manufacturing. Its cost is $10,000 and it is assumed to reduce the variance of the cost estimate seventy percent. This would generate a reduction in the profit variance of $100,000, and increase the utility by .07 units.

The increases described in the last paragraph would occur if the respective test was first in the information sequence. To supply the input necessary for analyzing study sequences, the effects of each study occurring in permutation with the other studies must be determined. In the example

cited earlier, test one produced a gain of .65 percent if it occurred after test two and test two produced a gain of .10 after test one. Test three would have little effect after test one, since the one city test would monitor little that the larger test had not already measured. The rejection level of the smaller test would have no effect on the expected value after the larger test and little or no reduction in the variance would be produced. If the smaller test market were done first, it would reduce the effectiveness of the larger test, since it had already gained some sales information. In this example, the large test's effectiveness is reduced to a utility increase of .47 units if the percentage change in variance and rejection percentage are considered to be unchanged. The cost test is independent of the others so its position in the network is unrestricted. The pricing study and the advertising study overlap so their effectiveness varies with their positions. The incremental reward of each possible second test in the sequence is described in Table 1. The effectiveness is measured by the utility. This process must be continued until the incremental gains of each test in each sequence have been calculated. With this information the optimum sequence can be found by the dynamic programming recursion in Equation (2). Since the cost study rewards are independent of other studies, it need not be included in the sequence specification. It can be placed in any position without changing the optimal reward. The complete example set of input is given in Table 1. The last stage inputs are merely the values of the remaining study that is needed to complete the network, so they can be added to S_3, d_3 matrix to form S_3', d_3'.

By use of the dynamic programming recursion in Equation (2), the maximum change in utility is 1.03 and the optimal sequence is 3, 4, 2, 1 or 3, 2, 4, 1 for a four test alternative. If a three test alternative is present, the

		J: 1	2	3	4
S_1, d_1	**Start**	.71	.26	.38	.26
S_2,d_2	I				
	1	---	.10	0	.10
	2	.65	---	.20	.21
	3	.47	.15	---	.17
	4	.51	.20	.21	---
S_3,d_3/S_3' ,d_3'	I				
	12	---	---	0/.05	.09/.09
	13	---	.08/.13	---	.07/.11
	14	---	.09/.09	0/.04	---
	21	---	---	0/.05	.09/.09
	23	.42/.47	---	---	.15/.50
	24	.46/.46	---	.15/.50	---
	31	---	.08/.13	---	.07/.11
	32	.42/.47	---	---	.15/.50
	34	.40/.44	.13/.48	---	---
	41	---	.09/.09	0/.04	---
	42	.46/.46	---	.15/.50	---
	43	.40/.44	.13/.48	---	---

TABLE I

INCREMENTAL UTILITY GAINS RESULTING FROM A DECISION
TO USE TEST J IN SEQUENCE I AT STATE S_k

optimal sequence is 2, 1, 4 with a change of 1.00 units. The best two test sequence is 2, 1 which produces a utility gain of .91 units. The cost study could be conducted at any point in the network with a gain of .07 so it could be added to the 3, 4, 2, 1 or 3, 2, 4, 1 sequence to produce a total gain in utility of 1.10 units. Since the objective of the information discounting is to maximize utility, the five test sequence should be chosen. The one city test should be conducted first, then the price or advertising study, and finally the five city test market. The total utility of the project is 7.50 after the information discounting.

Product Selection

After all the proposed projects have been subjected to such an information discounting, they can be arrayed and the best sub-set can be selected. Table Two summarizes the utilities, expected returns, standard deviations, and investment in terms of financial and managerial commitments. The utility is calculated by equation (4). The problem now is to select the best subset of these projects subject to the constraints on the firm. For this example, a financial constraint of twenty million dollars and a top managerial constraint of 300 man-weeks are assumed. The objective function for the example is prescribed in equation (3). The second and third order product interdependencies and covariances are given in Table Three. The expected value interdependencies indicate the 3D projector and Video Tape TV are substitutes. This is logical since both products serve a similar home entertainment need. The 3D projector and the wall TV are also substitutes, but the interdependency is less severe. The wall TV and the video tape recorder are complementary. This may be due to a production efficiency that could be gained by common usage of the same

Product	Utility	Expect Return	Std. Dev. of Profit	Investment Financial ($ mm)	Investment Managerial (man-weeks)
1-3D Projector	7.50	8.0	2.24	4.9	150
2-Video Tape Recorder	5.96	6.0	.6	4.5	75
3-Wall TV	12.98	14.0	3.2	9.8	90
4-TV Telephone	5.9	5.0	1.0	3.6	120
5-Programmable Calculator	9.77	10.0	1.5	5.9	100

TABLE TWO

INDIVIDUAL PROJECT RETURNS AND COMMITMENTS

Products i	j	μ_{ij} ($ mm)
1	2	-2.0
1	3	-1.0
1	4	+1.0
1	5	0
2	3	+6.0
2	4	+4.0
2	5	0
3	4	+2.0
3	5	0
4	5	0

Products i	j	k	μ_{ijk} ($mm)
1	2	3	-1.0
1	2	4	+2.0
1	2	5	0
1	3	4	+1.0
1	3	5	0
1	4	5	0
2	3	4	+3.0
2	3	5	0
2	4	5	0
3	4	5	0

σ_{ij}	1	2	3	4	5
1	5.0	.27	.72	.68	0
2	.27	.36	1.34	.30	0
3	.72	1.34	10.2	1.28	0
4	.68	.3	1.28	1.0	0
5	0	0	0	0	2.25

TABLE THREE

EXPECTED VALUE AND UNCERTAINTY INTERDEPENDENCIES

electronic components or a demand complementarity reflected in a preference and interest in TV based conviences. Product combinations two-four and one-four also display complementarity. Product five (the programmable calculator) is independent of the other products, because it serves essentially an industrial market. The covariances indicate all the product's fluctuations are positively correlated, except for the programmable calculator which is independent.

To maximize the utility of the selected subset as prescribed in equation (3), a trial and error technique was utilized. In this case the search space was so small that enumeration was possible. In larger problems where enumeration is infeasible, an on-line man-machine trial and error search routine[37] or other generalized search routine could be used. The results of the search are reported in Table Four. The subset of projects that maximize the utility subject to the investment contraint of twenty million dollars and 300 man-weeks of management time is the video tape recorder, wall TV and TV telephone. This combination has a total utility of 38.26 units and is greater than many combinations of four products. In fact, it is 13.59 units greater than the next feasible subset of projects (2 and 3). The three products are desirable because of their complementarity (see Table Two), and efficient resource utilization.

This example has demonstrated the proposed information discounting technique and interdependent project selection. The selection procedure assumed the projects were integer. If partial commitments were allowed for each project, the selection would have been more complex. The example also assumed that the optimum marketing program for each project that would have been specified in the first stage of the analysis remained optimal.

3D Projector	Video Tape Recorder	Wall TV	TV Telephone	Calculator	UTILITY	MANAGEMENT COMMITMENT (MAN-WEEK)	INVESTMENT (MM $)
X	X	X	X	X	55.20	535	28.7
	X	X	X	X	48.79	385	23.8
X	X	X	X		45.42	435	22.8
	X	X	X		38.26	285	17.9
X	X	X		X	38.78	415	25.1
X		X	X	X	37.62	460	24.2
	X	X		X	34.45	265	20.2
X	X		X	X	32.88	445	18.9
X	X	X			29.97	315	19.2
		X	X	X	29.40	310	19.3
X		X		X	29.11	340	20.6
X		X	X		27.84	360	18.3
	X	X			24.67	165	14.3
	X		X	X	24.58	295	14.0
	⋮				⋮	⋮	⋮
		X		X	22.75	190	15.7
		X	X		19.62	210	13.4
X		X			19.33	240	14.7
X				X	17.27	250	10.8
	X			X	15.74	175	10.4
	X		X		14.80	190	8.1
			X	X	14.67	220	9.5
		X			12.98	90	9.8

TABLE FOUR

SOME PRODUCT SELECTION ALTERNATIVES

This is not generally true. Each possible combination theoretically should be re-optimized by a first stage model such as SPRINTER to re-specify the optimum levels for the controllable marketing, production, and finance variables for the proposed combination of product selections.

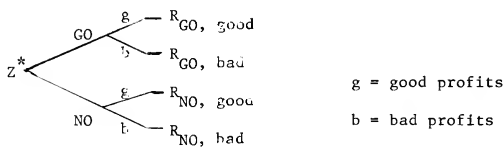
SUMMARY

This paper has developed a model for use in new product selection. Given that the relevant controllable variables of the new products have been specified for maximum profit, the model used an "information discounting" procedure to attribute the maximum gains that could accrue to each project by optimally transversing an information network. This discounting is based on the perception of two information functions. The first is to reduce uncertainty and is measured by a reduction in the variance of the estimates. The second function is to prevent the mistake of introducing a poor product and is measured by the increase in expected value generated by establishing a rejection level for a test. These two changes were imputed into one utility surrogate which served to measure the benefits of information studies. After specifying the conditional effects of experimental design in the proposed information network, dynamic programming was used to identify the optimum information sequence. When the optimum sequence rewards were added to the current designation of the product's utility,quadratic, integer, and heuristic programming techniques for selection from the set of proposals were discussed with respect to their ability to comprehend product interdependency and uncertainty. A hypothetical example of a new product selection problem and solution was presented.

APPENDIX - THE HEURISTIC IN BAYESIAN TERMS

The proposed heuristic procedure can be expressed in terms of Bayesian pre-posterior analysis. The change in the standard deviation can be thought of as the change in the variance of the distribution that would result from the application of a particular design of an information study. This change is reflected in the variance of the pre-posterior distribution. The manager is called upon to estimate these effects on the basis of past experiences with the type of experimental design.

The expected value gains described in the heuristic are generated by establishing a product rejection level for the information study. The best rejection level is the value of the study that would be associated with a rejection decision in the basis of Bayesian pre-posterior analysis. For example, let us assume only two decision alternatives are present: GO and NO. If a study measures sales, the rejection level for the test should be a value below which a NO decision would be best. The problem is to specify a value for the test study Z^* that bounds the area in which a NO decision should be made. Since we are concerned only with GO and NO, let us consider only two states of nature: good and bad. The decision tree branch at Z^* is described in Figure A-1 with the payoffs described by $R_{k,j}$



DECISION TREE AT REJECTION LEVEL (Z^*)
FIGURE A-1

If Z^* is correctly chosen, then the utility of the NO decision must be greater than or equal to the utility of the GO decision.

Mathematically

$$\text{(a-1)} \quad P(\text{good}|Z^*)\, R_{GO,good} + P(\text{bad}|Z^*)\, R_{GO,bad} \leq P(\text{good}|Z^*)\, R_{NO,good} + P(\text{bad}|Z^*) P_{NO,ba}$$

where

$$\text{(a-2)} \quad P(\text{good}|Z^*) = \frac{P(Z^*|\text{good})\, P(\text{good})}{P(Z^*)}$$

and

$$\text{(a-3)} \quad P(\text{bad}|Z^*) = \frac{P(Z^*|\text{bad})\, P(\text{bad})}{P(Z^*)}$$

when (a-2) and (a-3) are substituted in (a-1) and simplified

$$\frac{P(Z^*|\text{good})\, P(\text{good})}{P(Z^*)} \left(R_{GO,good} - R_{NO,good}\right) \leq \frac{P(Z^*|\text{bad})\, P(\text{bad})}{P(Z^*)} \left(R_{NO,bad} - R_{GO,bad}\right)$$

or

$$\text{(a-4)} \quad P(Z^*|\text{good})\, P(\text{good})\left(R_{GO,good} - R_{NO,good}\right) \leq P(Z^*|\text{bad})\, P(\text{bad})\left(R_{NO,bad} - R_{GO,bad}\right)$$

Now, given payoff values and prior probabilities, the values of Z^* can be found by examining the estimates of the conditional probabilities: $P(Z_j|\text{good})$ and $P(Z_j|\text{bad})$. The lowest Z's would be examined first and then greater values would be examined until condition (a-2) was violated. The test value where this occurs is Z^* and the best value for the test rejection level.

Although it is possible to express the basic nature of the heuristic in Bayesian terms, it is difficult to explicitly trace the analytic behavior in a sequential analysis. Indeed if one could do this there would be no need for the heuristic since the basic Bayesian analysis is clearly superior in theory. The heuristic is used to reduce the computational burden of the

sequential analysis. It is an alternative to pruning the tree and may be useful in many classes of complex decision tree analyses. Instead of cutting the decision tree branches, the number of alternatives is reduced by aggregating the branches and supplying input about experimental designs in the form of a rejection level and an estimate of the variance reduction.

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